LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **MATHEMATICS**

FIFTH SEMESTER – NOVEMBER 2011

MT 5406 - COMBINATORICS

Date : 12-11-2011 Dept. No. Max. : 100 Marks
Time : 9:00 - 12:00

SECTION-A

ANSWER **ALL** THE QUESTIONS: **10 x 2 = 20**

1. Define stirling no of I kind.
2. Draw the Ferrers graph for $\lambda = (544311)$. Also find λ'.
3. Define a binomial number.
4. Give the value of the complete homogenous symmetric function h_4.
5. How many ways one can move from the point (0,0,0) to (a,b,c) through neighbouring points.
6. State Cauchy's theorem.
7. Find the permanent of $\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$.
8. State Euler function.
9. When does a circular word have primitive period?.
10. Define G-equivalence.

SECTION B

ANSWER ANY **FIVE** QUESTONS: **5 x 8 = 40**

11. Show that there exists a bijection between the following two sets:
    (a) The set of *n*-tuples on *m* letters without repetition.
    (b) The set of injections of an *n*-set into an *m*-set.
    Prove that the cardinality of each of these sets is m (m-1) (m-2) … (m-n+1).
12. Write down the partition of 2,3,4,5 and derive the recurrence formula for P_n^m.
13. Prove that the elements *f* of *R[t]* given by $f = \sum_{k=0}^{\infty} a_k t^k$ has an inverse in *R[t]* if and only if α_0 has an inverse in *R.*
14. Describe the Monomial symmetric function and the Complete homogenous symmetric function.
15. State and prove multinomial theorem on a commutative ring *A*.
16. Derive sieve formula. How many numbers between 1 and 100 are not divisible by 2,3 or 5.
17. There are 20 marbles of the same size but of different colors 1 red, 2 blue, 2 green, 3 white, 3 yellow, 4 orange and 5 black in an urn. Find the number of ways of arranging 5 Marbles from this urn in a row.
18. Define rook polynomial and show that $R(t,c) = t\,(R(t, C_{dd})) + R(t, C_d^1)$.

SECTION C

ANSWER ANY **TWO** QUESTIONS: **2x20 = 40**

19. (a)Prove that the cardinality of the set of combination of m symbols taken n at a time with repetition permitted is $\frac{[m]^n}{n!}$.

    (b)Prove that the number of arrangements n distinct objects into m distinct boxes with the objects in each box arranged in a definite order is $[m]^n$. (10+10)

20. (a)Derive the recurrence formula for S_n^m. Tabulate the values of S_n^m for *n, m* = 1, 2, 3, 4, 5. And find the Bell number.

    (b) Explain in detail the power sum symmetric functions. (10+10)

21. (a) State and prove generalized inclusion and exclusion principle.

    (b) How many 4-letter words with distinct letters can be got from the word UNIVERSAL?

    (c) With proper illustrations describe the problem of Fibonacci. (8+2+10)

22. (a)State and solve the ménage problem.

    (b)State and prove the Burnside's lemma. (10+10)

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